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(Pages : 3)

MAT I—401

FOURTH SEMESTER B.E. DEGREE EXAMINATON

ENGINEERING MATHEMATICS—IV

(B.E. Common to E & C, IT etc.)

19

Time : Three Hours | Maximum : 100 Marks

Answer any **five** *questions.*

All questions carry equal marks.

Statistical tables are allowed.

1. (a) Obtain $J_n(x)$ as a solution of the Bessel's differential equation :

$$x^2y'' + xy' + (x^2 - n^2)y = 0.$$

(7 marks)

(b) Show that : $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ and $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. (3 + 3 = 6 marks)

(c) Prove that :

(i) $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$.

(ii) $J_{-n}(x) = (-1)^n J_n(x)$.

(4 + 3 = 7 marks)

2. (a) (i) Prove the Rodrigue's formula :

$$P_n(x) = \frac{1}{2^n (n!)} \frac{d^n}{dx^n}\left\{(x^2 - 1)^n\right\}.$$

(ii) Find the value of $P_n(0)$.

(5 + 2 = 7 marks)

(b) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. (6 marks)

(c) Prove that :

$$\int_{-1}^{+1} P_m(x) \cdot P_n(x)\, dx = 0 \text{ for } m \neq n$$

$$= \frac{2}{2n+1} \text{ for } m = n.$$

(7 marks)

3. (a) Define the following with illustrations :—

(i) Objective function.

(ii) Feasible solution.

(iii) Optimization.

(iv) Slack and surplus variables.

(6 marks)

(b) By using graphical method, maximize :

$Z = x_1 - 3x_2$, where $x_1 \geq 0$, $x_2 \geq 0$ subject to the conditions : $3x_1 + 4x_2 \geq 19$, $2x_1 - x_2 \leq 9$, $2x_1 + x_2 \leq 15$ and $x_1 - x_2 \geq 3$.

(7 marks)

Turn over

(c) Use the Simplex method to maximize $Z = 3x + 4y$ subject to the constraints $2x + y \le 40$, $2x + 5y \le 180$ and $x \ge 0, y \ge 0$.

(7 marks)

4. (a) Find all the basic solutions of the following systems of equations identifying in each case the basic and non-basic variables :

$$2x_1 + x_2 + 4x_3 = 11,\ 3x_1 + x_2 + 5x_3 = 14.$$

(7 marks)

(b) Express an L.P.P. in (i) Canonical form ; (ii) Standard form. (6 marks)

(c) Using artificial variable method, minimize $Z = 2x_1 + x_2$ subject to $3x_1 + x_2 = 3$; $4x_1 + 3x_2 \ge 6$; $x_1 + 2x_2 \le 3$; $x_1 \ge 0, x_2 \ge 0$.

(7 marks)

5. (a) Find the 3 quartiles for the following distribution :—

Class	0—10	10—20	20—30	30—40	40—50	50—60	60—70	70—80	80—90	90—100
Frequency	5	13	12	11	8	4	1	3	1	2

(6 marks)

(b) The mean and mode of the following wage distribution of 230 persons are known to be Rs. 33.5 and Rs. 34 respectively. Find the missing frequencies :

Wage in Rs. :	0—10	10—20	20—30	30—40	40—50	50—60	60—70
Frequency :	4	16	—	—	—	—	—

(7 marks)

(c) The following are the scores made by two cricketers in 10 innings. Find which of the two cricketers is a better score on an average ? Also find which of them is more consistent :

A :	31	48	13	51	38	43	50	36	47	82
B :	51	5	12	83	37	112	42	18	79	0

(7 marks)

6. (a) With the usual notation, prove that :

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and extend this result to 3 events A, B, C.

(7 marks)

(b) (i) A problem is given to three students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$ respectively. Find the probability that the problem is solved.

(ii) If $A \subset B$; prove that $P(A) \le P(B)$.

(4 + 3 = 7 marks)

(c) State and prove Baye's theorem.

(6 marks)

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7. (a) The probability distribution of a finite random variable X is given by the following table :—

x_i	:	−2	−1 0	1	2	3
$P(x_i)$	:	0.1	K 0.2	2 K	0.3	K

Find the values of K, mean and variance.

(6 marks)

(b) When a coin is tossed 4 times, find the probability of getting : (i) exactly one head ; (ii) at most 3 heads ; and (iii) at least two heads.

(7 marks)

(c) The probability that an individual suffers a bad reaction from a certain injection is 0.002. Determine the probability that out of 1000 individuals (i) exctly 3 ; (ii) more than 2 will suffer a bad reaction.

(7 marks)

8. (a) Define the following :—

(i) Stochastic process ; (ii) Stationary process.

(6 marks)

(b) Find the auto correlation $R(t_1, t_2)$ of the stochastic process defined by

$$X(t) = A \cos(\omega t + \alpha)$$

where the random variables A and α are independent, and α is uniform in the interval $[-\pi, \pi]$.

(7 marks)

(c) Define : (i) Ergodicity in the mean ; (ii) Ergodicity in the mean square ; (iii) Ergodicity in the correlation function. Also discuss the Ergodic property in the mean of the R.T.S.

(7 marks)

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FOURTH SEMESTER B.E. DEGREE EXAMINATON

ENGINEERING MATHEMATICS—IV

(B.E. Common to E & C, IT etc.)

23

Time : Three Hours Maximum : 100 Marks

Answer any five questions.
All questions carry equal marks.
Statistical tables are allowed.

1. (a) Obtain $J_n(x)$ as a solution of the Bessel's differential equation :

$$x^2y'' + xy' + (x^2 - n^2)y = 0.$$

(7 marks)

(b) Show that : $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ and $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. (3 + 3 = 6 marks)

(c) Prove that :

(i) $\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$.

(ii) $J_{-n}(x) = (-1)^n J_n(x)$.

(4 + 3 = 7 marks)

2. (a) (i) Prove the Rodrigue's formula :

$$P_n(x) = \frac{1}{2^n (n!)} \frac{d^n}{dx^n}\left\{(x^2 - 1)^n\right\}.$$

(ii) Find the value of $P_n(0)$.

(5 + 2 = 7 marks)

(b) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. (6 marks)

(c) Prove that :

$$\int_{-1}^{+1} P_m(x) \cdot P_n(x)\, dx = 0 \text{ for } m \neq n$$

$$= \frac{2}{2n+1} \text{ for } m = n.$$

(7 marks)

3. (a) Define the following with illustrations :—

(i) Objective function. (ii) Feasible solution.

(iii) Optimization. (iv) Slack and surplus variables.

(6 marks)

(b) By using graphical method, maximize :

$Z = x_1 - 3x_2$, where $x_1 \geq 0$, $x_2 \geq 0$ subject to the conditions : $3x_1 + 4x_2 \geq 19$, $2x_1 - x_2 \leq 9$, $2x_1 + x_2 \leq 15$ and $x_1 - x_2 \geq 3$.

(7 marks)

Turn over

(c) **Use the Simplex method to maximize $Z = 3x + 4y$ subject to the constraints $2x + y \le 40$, $2x + 5y \le 180$ and $x \ge 0, y \ge 0$.**

(7 marks)

4. (a) Find all the basic solutions of the following systems of equations identifying in each case the basic and non-basic variables :

$$2x_1 + x_2 + 4x_3 = 11,\ 3x_1 + x_2 + 5x_3 = 14.$$

(7 marks)

(b) Express an L.P.P. in (i) Canonical form ; (ii) Standard form. (6 marks)

(c) Using artificial variable method, minimize $Z = 2x_1 + x_2$ subject to $3x_1 + x_2 = 3$; $4x_1 + 3x_2 \ge 6$; $x_1 + 2x_2 \le 3$; $x_1 \ge 0, x_2 \ge 0$.

(7 marks)

5. (a) Find the 3 quartiles for the following distribution :—

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Frequency	5	13	12	11	8	4	1	3	1	2

(6 marks)

(b) The mean and mode of the following wage distribution of 230 persons are known to be Rs. 33.5 and Rs. 34 respectively. Find the missing frequencies :

Wages in Rs. :	0—10	10—20	20—30	30—40	40—50	50—60	60—70
Frequency :	4	16	—	—	—	—	—

(7 marks)

(c) The following are the scores made by two cricketers in 10 innings. Find which of the two cricketers is a better score on an average ? Also find which of them is more consistent :

A :	31	48	13	51	38	43	50	36	47	82
B :	51	5	12	83	37	112	42	18	79	0

(7 marks)

With the usual notation, prove that :

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and extend this result to 3 events A, B, C.

(7 marks)

A problem is given to three students A, B, C whose chances of solving it are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively. Find the probability that the problem is solved.

If $A \subset B$; prove that $P(A) \le P(B)$. (4 + 3 = 7 marks)

State and prove Baye's theorem. (6 marks)

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7. (a) The probability distribution of a finite random variable X is given by the following table :—

x_i	:	-2	$-1\ 0$	1	2	3
$P(x_i)$	:	0.1	K 0.2	2 K	0.3	K

Find the values of K, mean and variance.

(6 marks)

(b) When a coin is tossed 4 times, find the probability of getting : (i) exactly one head ; (ii) at most 3 heads ; and (iii) at least two heads.

(7 marks)

(c) The probability that an individual suffers a bad reaction from a certain injection is 0.002. Determine the probability that out of 1000 individuals (i) exctly 3 ; (ii) more than 2 will suffer a bad reaction.

(7 marks)

8. (a) Define the following :—

(i) Stochastic process ; (ii) Stationary process.

(6 marks)

(b) Find the auto correlation $R(t_1, t_2)$ of the stochastic process defined by

$$X(t) = A\cos(\omega t + \alpha)$$

where the random variables A and α are independent, and α is uniform in the interval $[-\pi, \pi]$.

(7 marks)

(c) Define : (i) Ergodicity in the mean ; (ii) Ergodicity in the mean square ; (iii) Ergodicity in the correlation function. Also discuss the Ergodic property in the mean of the R.T.S.

(7 marks)

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MAT 41

Fourth Semester B. E. Examination
NEW SCHEME
Engineering Mathematics-IV
(common to all branches)

Model Question Paper-2

Time: 03 Hours **Max. Marks: 100**

Note: Answer any **FIVE** full questions, choosing at least one question from each part.
All questions carry equal marks.

PART - A

1. a) Show that $f(z) = z+e^z$ is analytic and hence find $f^1(z)$.
 b) Determine the analytic function f(z)=u+iv given that $u = e^{2x}\{x \cos 2y - y \sin 2y\}$.
 c) Define conformal transformation. Discuss the transformation $w = e^z$.
 (7+7+6=20 Marks)

2. a) State and prove Cauchy's theorem.
 b) Find the Laurent series of $f(z) = \dfrac{1}{(z-1)(z-2)}$ in the regions i) $|z|>2$; ii) $1<|z|<2$.,
 c) Determine the poles of the function $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ and residue at each pole. **(7+7+6=20 Marks)**

PART - B

3. a) Obtain the series solution of Bessel's differential equation in the form $y = A\,J_n(x)+B\,J_{-n}(x)$.
 b) Prove that $J_n(x) = 1/\pi \int_0^\pi \cos(n\theta - x\sin\theta)\,d\theta$.
 c) Prove the recurrence relation $d/dx\,[x^n J_n(x)] = x^n J_{n-1}(x)$. **(7+7+6=20 Marks)**

4. a) Prove that $xP'_n(x) - P'_{n-1}(x) = nP_n(x)$.
 b) Prove that $\int_{-1}^{+1} P_m(x)\,P_n(x)\,dx = \begin{cases} 0, & \text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n \end{cases}$
 c) Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre Polynomials.
 (7+7+6=20 Marks)

PART – C

5. a) Fit a parabola of the form $y = a + bx + cx^2$ for the following data by the method of least squares.

x :	0	1	2	3	4
y :	1.0	1.8	1.3	2.5	2.3

 b) Fit a regression line of y on x for the following data:

x :	10	14	18	22	26	30
y :	18	12	24	06	30	36

 c) State and prove Bayes' theorem on conditional probability. **(7+7+6=20 Marks)**

Contd.....2.

6. a) Define random variables and classify them. The p.d.f. of a variate x is given by the following table.

X:	0	1	2	3	4	5	6
Y:	K	3K	5K	7K	9K	11K	13K

For what value of K, this represents a valid probability distribution? Also find i) $P(x \geq 5)$ and ii) $P(3 < x \leq 6)$

b) Find the mean and variance of Poisson distribution.

c) In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for i) 10 minutes or more ii) Less than 10 minutes iii) Between 10 and 12 minutes.

(7+7+6=20 Marks)

PART - D

7. a) Explain the following terms :
   i) Type I and Type II errors
   ii) Null hypothesis
   iii) Level of significance

b) A coin is tossed 1000 times and it turn up head 540 times. Decide on the hypothesis that the coin is un-biased.

c) Ten individuals are chosen at random from a population and their heights in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that the mean height of the universe is 66 inches. {value of $t_{0.05}$ = 2.262 for 9 d.f}

(7+7+6=20 Marks)

8. a) The joint distribution of two random variables x and y is given by the following table.

X \ Y	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Determine the marginal distribution of x and y. Also, verify that x and y are independent.

b) Assume that a computer system is in one of the three states: busy, idle or undergoing repair denoted by states 0, 1, 2. Observing its state at a certain specified time on each day, it is found that the system approximately behaves like a Markov chain with the transition probability matrix

$$\begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{pmatrix}$$

Prove that the chain is irreducible, and determine the steady-state probabilities.

c). Define stochastic matrix. Find the unique fixed probability vector for the regular stochastic matrix:

$$\begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$$

(7+7+6=20 Marks)

-END-

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MATL401

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Fourth Semester B.E. Degree Examination, August 2001

EC/TE/IT/BM/ML

Engineering Mathematics - IV

Time: 3 hrs.] 18 [Max.Marks : 100

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Note: *Answer any FIVE full questions.*
Statistical tables are allowed

1. (a) Show that $J_0^2(x) + 2J_1^2(x) + 2J_2^2(x) + 2J_3^2(x) + \ldots\ldots = 1$

(b) Prove that $2J_n^1(x) = J_{n-1}(x) - J_{n+1}(x)$

(c) If α and β are the roots of the equation $J_n(x) = 0$. Then show that
$\int_0^1 xJ_n(\alpha x)J_n(\beta x)dx = \frac{1}{2}[J_n^1(\alpha)]^2$ for $\alpha = \beta$

2. (a) Express the polynomial $x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials

(b) Prove that (i) $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$
(ii) $P_n(1) = 1$ and $P_n(-x) = (-1)^n P_n(x)$

(c) Prove that $\int_{-1}^{+1} P_m(x)P_n(x)dx = 0$ for $m \neq n$

3. (a) Explain briefly the classification of optimization problems

(b) Solve the LPP graphically. Maximize $Z = 3x_1 + 4x_2$ subject to the constraints
$4x_1 + 2x_2 \leq 80$
$2x_1 + 5x_2 \leq 180$ and $x_1, x_2 \geq 0$

(c) Using Simplex method, maximize $Z = x_1 + 3x_2$.
subject to $x_1 + 2x_2 \leq 10$ $x_1 \leq 5$, $x_2 \leq 4$ and $x_1, x_2 \geq 0$

4. (a) Define the following terms
i) Slack and Surplus variables
ii) Optimum solution
iii) Degeneracy in simplex method

(b) Using Simplex method
Maximize $Z = 2x_1 + 4x_2 + 3x_3$ subject to the constraints
$3x_1 + 4x_2 + 2x_3 \leq 60$
$2x_1 + x_2 + 2x_3 \leq 40$
$x_1 + 3x_2 + 2x_3 \leq 80$ and $x_1, x_2, x_3 \geq 0$

5. (a) Show that the variance of the first n positive integers is $\frac{n^2-1}{12}$

(b) Compute the mean, median and standard deviation for the following

x:	1-10	11-20	21-30	31-40	41-50	51-60
f:	3	16	26	31	16	8

Contd.... 2

(c) The mean and standard deviation of 21 observations are 30 and 5 respectively. It was subsequently noted that one of the observations, namely 10 was not correct. Omit it and determine the mean and standard deviation of the rest.

6. (a) If the events A and B are independent then show that $P(A \cap B) = P(A) \cdot P(B)$

(b) Let A and B be events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$. Find i) $P(A/B)$ ii) $P(B/A)$ iii) $P(\bar{A}/\bar{B})$ iv) $P(\bar{B} / \bar{A})$

(c) A bag X contains 2 white, 3 red balls and a bag Y contains 4 white, 5 red balls. One ball is drawn at random from one of the bags, and is found to be red. Find the probability that it was drawn from bag Y.

7. (a) A random variable X has the following probability function Values of x :

x:	0	1	2	3	4	5	6	7
P(x):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

i) find k(ii) Evaluate $P(x < 6)$, $P(x \geq 6)$ and $P(3 < x \leq 6)$

(b) A random variable x has the following density function

$$P(x) = \begin{cases} kx^2, & -3 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

Evaluate k, and find (i) $P(1 \leq x \leq 2)$ (ii) $P(x \leq 2)$ (iii) $P(x > 1)$

(c) Derive the expressions for mean and standard deviation in the case of binomial distribution.

8. (a) Define a Stochastic process and classify the various types of Stochastic processess.

(b) Find the auto correlation $R(t_1, t_2)$ of the Stochastic process defined by $x(t) = A\cos(wt + \alpha)$ where the random variable A and α are indepedent and α is uniform in the interval $[-\pi, \pi]$

(c) Define

i) Ergodicity in the mean

ii) Ergodicity in the mean square

iii) Ergodicity in the correlation function.

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Fourth Semester B.E. Degree Examination, February 2002

EC/TE/IT/BM/ML

Engineering Mathematics - IV

Time: 3 hrs.] [Max.Marks : 100

Note: *Answer any FIVE full questions. Statistical tables are allowed*

1. (a) If n is a positive integer prove that
$J_n(x) = \frac{1}{\pi}\int_0^{\pi} Cos(n\theta - x Sin\theta)d\theta$ **(8 Marks)**

(b) Prove that i) $J_0^2 + 2[J_1^2 + J_2^2 + J_3^2 + ...\infty] = 1$

ii) $J_n(-x) = (-1)^n J_n(x)$ **(4+4=8 Marks)**

(c) Prove that $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$ **(4 Marks)**

2. (a) Express $f(x) = x^3 + 2x^2 - 4x + 5$ interms Legendre's polynomials. **(6 Marks)**

(b) Derive the Rodrigues formula

$$P_n(x) = \frac{1}{2^n \angle n}\frac{d^n}{dx^n}[(x^2-1)^n]$$

(7 Marks)

(c) Show that $\int_{-1}^{1} x^2 P_{n-1}(x).P_{n+1}(x)dx = \frac{2n(2n+1)}{(2n-1)(2n+1)(2n+3)}$ **(7 Marks)**

3. (a) Define the terms : i) objective function ii) slack variables iii) surplus variables **(2+2+2=6 Marks)**

(b). Use the graphical method to minimize $Z = 20x + 16y$ subject to the constraints $3x + y \geq 6, x + y \geq 4, \ x + 3y \geq 6$ and $x, y \geq 0$ **(7 Marks)**

(c) Obtain the mathematical format of the following LPP . A manufacturer produces two types of models M_1 and M_2. Each M_1 model requires 4 hours of grinding and 2 hours of polishing; where as each M_2 model requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works for 60 hours of a week. profit on M_1 model is Rs 3 and on M_2 model is Rs 4. Whatever is produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models so that he may make the maximum profit in a week. **(7 Marks)**

4. (a) Use the simplex method to maximize $P = 5x + 8y$ subject to the constraints $4x + 6y \leq 24, \ 2x + y \leq 18, \ 3x + 9y \leq 36$ and $x \geq 0, y \geq 0$. **(10 Marks)**

(b) Use the simplex method to minimize $P = x - 3y + 2z$ subject to the constraints $3x - y + 2z \leq 7, \ -2x + 4y \leq 12, \ -4x + 3y + 8z \leq 10, \ x \geq 0, y \geq 0, z \geq 0$. **(10 Marks)**

5. (a) The mean of five items of an observation and the variance are 4 and 5.2. If three of the items are 1,2 and 6, then find the other two. **(6 Marks)**

Contd.... 2

(b). If m, n are the items, $\overline{x}$ and $\overline{y}$ are the AMs and σ_1, σ_2 be the S.D's of two sets of data then prove that the standard deviation of the combined data is

$$\sigma^2 = \frac{1}{m+n}[m\sigma_1^2 + n\sigma_2^2 + \frac{mn}{m+n}(\overline{x} - \overline{y})^2]$$

(7 Marks)

(c) Find the mean deviation from the mean and standard deviation of $A.P$: $a, a+d, a+2d, ..., a+nd, ...a+2nd$ and verify that the later is greater than the former. **(7 Marks)**

6. (a) Define conditional probability and independent events. Prove that if A, B are independent events then $\overline{A}$ and $\overline{B}$ are also independent events. **(6 Marks)**

(b) Given a binary communication channel where A is the input and B is the output, let $P(A) = .4, P(B/A) = .9$ and $P(\overline{B}/\overline{A} = 0.6$. Find i) $P(A/B)$ ii) $P(A/\overline{B})$ **(6 Marks)**

(c) In a binary communication channel, data is transmitted through 3 digit code. But due to noise, there is a chance that the transmitted digit 0 is received as 1 with probability .06 and the transmitted digit 1 is received as 0 with probability .09. At a given point of time what is the probability i) a 3 digit code, transmitted is received correctly if the probability of transmitting 0 is .45 ii) a code transmitted as 101 is received as 011. **(8 Marks)**

7. (a) A random variable X has the following probability function.

X	-2	-1	0	1	2	3
p(x)	.1	k	0.2	2k	0.3	k

Find i) the value of k ii) Mean iii) variance iv) $P(x < 2)$ v) $p(x > 2)$ vi) $p(-1 \leq x < 3)$ **(6 Marks)**

(b) Define moment generating function of the probability distribution. Find the mgf of the distribution $f(x) = \frac{1}{c}e^{-x/c}, (x > 0), c > 0$ hence find its mean and standard deviation. **(6 Marks)**

(c) i) The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped, find the probability that i) exactly two will strike the target ii) atleast two will strike the target. **(4 Marks)**

ii) If a random variable has Poisson distribution such that $(P(1) = P(2)$, find the mean of the distribution and hence find $P(4)$. **(4 Marks)**

8. (a) Define a stochastic process and classify the types of stochastic processes with an example each. **(8 Marks)**

(b) Define the Random telegraph signal process. Show that its mean is zero and the auto correlation function and auto covariance are equal. **(6 Marks)**

(c) Define the term ergodicity with respect to mean.
If the random process $X(t) = ACos wt + BSin wt$ Where A and B are uncorrelated random variables each with mean zero and variance 1 and w is a positive constant, show that this process is ergodic in the mean **(6 Marks)**

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MATL401

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Fourth Semester B.E. Degree Examination, July/August 2003

EC/TE/IT/BM/ML

Engineering Mathematics - IV

Time: 3 hrs.] [Max.Marks : 100

Note: ***Answer any FIVE full questions.***

1. (a) Obtain the series solution to the Bessel's differential equation $x^2y'' + xy' + (x^2 - n^2)y = 0$ in the form $y = AJ_n(x) + BJ_{-n}(x)$. (8 Marks)

(b) Prove that $\int_0^1 xJn(\alpha x)J(\beta x)dx = 0$ where α and β are distinct roots of $Jn(x) = 0$. (6 Marks)

(c) Prove that

i) $2nJ_n(x) = x[J_{n-1}(x) + J_{n+1}(x)]$

ii) $J_n'(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$ (6 Marks)

2. (a) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. (7 Marks)

(b) Show that $P_n(x)$ is the coefficient of Z^n in the expansion of $(1 - 2xz + z^2)^{-\frac{1}{2}}$. (6 Marks)

(c) Show that

$$\int_{-1}^{1} P_n^2(x)dx = \frac{2}{2n+1}.$$ (7 Marks)

3. (a) Define : i) Slack variables ii) Surplus variables.

Convert the following L.P.P. to the standard form :

Maximize $Z = 3x_1 + 5x_2 + 7x_3$

Subject to $6x_1 - 4x_2 \le 5,\ 3x_1 + 2x_2 + 5x_3 \ge 11,$

$4x_1 + 3x_3 \le 2,\ x_1, x_2 \ge 0$ (6 Marks)

(b) Using graphical method, solve the following L.P.P :

Maximize $Z = 2x_1 + 3x_2,$

subject to $x_1 - x_2 \le 2,\ x_1 + x_2 \ge 4,\ x_1, x_2 \ge 0$ (7 Marks)

(c) Find all basic solutions and optimal basic solution for the following problem:

Maximize $Z = 2x + 3y + 4z + 7t$

subject to

$2x + 3y - z + 4t = 8,\ x - 2y - 6z - 7t = -3,$

$x \ge 0, y \ge 0, z \ge 0, t \ge 0$ (7 Marks)

4. (a) Using the simplex method

Maximize $Z = 5x_1 + 3x_2$

subject to $x_1 + x_2 \le 2, 5x_1 + 2x_2 \le 10$

$3x_1 + 8x_2 \le 12,\ x_1, x_2 \ge 0$ (10 Marks)

Contd.... 2

(b) Solve the following L.P.P. by simplex method:

Minimize $Z = x_1 - 3x_2 + 3x_3$

subject to $3x_1 - x_2 + 2x_3 \leq 7,\ 2x_1 + 4x_2 \geq -12,$

$-4x_1 + 3x_2 + 8x_3 \leq 10,\ x_1, x_2, x_3 \geq 0$ (10 Marks)

5. (a) The mean and mode of the following wage distribution of 230 persons ar known to be Rs. 33.5 and Rs. 34 respectively. Find the missing frequencies.

Wage in Rs.	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	16	?	?	?	6	4

(7 Marks)

(b) The items of a certain obeservation are in the Arithmetic progression $a, a+d, a+2d, \ldots\ldots a+2nd$. Find the mean deviation from the mean. (6 Mar

(c) The following are the runs scored by two batsmen A and B in 10 successive innings :

A :	12	115	6	73	7	19	119	36	84	29
B :	47	12	16	42	4	51	37	48	13	0

Find who is the better score-getter and who is more consistent. (7 Marks)

6. (a) Three machines A, B, C manufacture respectively 0.4, 0.5 and 0.1 of the total production. The defective items produced by A, B and C respectively are 2%, 4% and 1% respectively. For an item chosen at random, what is the proba that it is defective? (6 Ma s)

(b) Let A and B be independent events. Show that i) A and $\overline{B}$, and ii) $\overline{A}$ and $\overline{B}$ are independent. If A and B are not mutually exclusive, show that $\overline{A}$ and $\overline{B}$ are independent. (7 Ma ks

(c) There are 3 bags : first containing 1 white, 2 red, 3 green marbles; second white, 3 red and 1 green marbles and third 3 white, 1 red and 2 green mar es Two marbles are drawn from a bag chosen at random. These are found to b one white and one red. Find the probability that the marbles so drawn c m from the second bag. (7 Mark

7. (a) Define i) random variable ii) probability mass function iii) probability densit function iv) moment generating function for a discrete probability distribu o (8 Marl

(b) Find the mean and standard deviation of Binomial distribution. (6 Marl

(c) In a certain factory turning out razor blades, there is a small char 0.002 for any blade to be defective. The blades are supplied in packets 10. Use Poisson distribution to calculate the approximate number of containing no defective, one defective and two defective blades respectiv a consignment of 10,000 packets. (6 a

8. (a) If Y and Z are two independent random variables with zero mean and standard deviation σ, find the mean and auto correlation of the process $\{x(t)\}$ where $x(t) = y - tz$. (7 Ma

(b) Define the random signal process. Show that its mean is zero and th a correlation function and auto covariance are equal. (7 Ma

(c) Find the power spectrum of the random telegraph signal whose ACF is $R(\tau) = e^{-2\lambda|\tau|}$ where $\lambda > 0$. (1

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NEW SCHEME

MAT41

VIVEKANANDA COLLEGE OF ENGG. & TECHNOLOGY LIBRARY PUTTUR

USN | | | | | | | | | | |

Fourth Semester B.E Degree Examination, January/February 2005

Engineering Mathematics IV

Common to all branches

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least one question from each part.*
2. All questions carry equal marks.
3. Use of statistical tables allowed.

Part A

1. (a) Derive Cauchy-Riemann equations in Cartesian form. **(7 Marks)**

(b) Find an analytic function $f(z) = u + iv$, given that
$u = x^2 - y^2 + \frac{x}{x^2+y^2}$ **(7 Marks)**

(c) Find the bilinear transformation that maps the points $z = -1, i, +1$ onto the points $w = 1, i, -1$ respectively. **(6 Marks)**

2. (a) State and prove Cauchy's integral formula. **(7 Marks)**

b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ as Laurent series in the regions
i) $1 < |z| < 2$ ii) $|z| > 2$ **(7 Marks)**

(c) Evaluate $\int_C \frac{e^{2z}}{(z+1)^3} dz$ where c is $|z| = \frac{3}{2}$ by using Cauchy's residue theorem. **(6 Marks)**

Part B

3. (a) If α is a root of $J_n(x) = 0$, prove that
$\int_0^1 x J_n^2(\alpha x) dx = \frac{1}{2}\{J_n'(\alpha)\}^2$ **(7 Marks)**

(b) Prove that
i) $2nJ_n(x) = x\{J_{n-1}(x) + J_{n+1}(x)\}$
ii) $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$ **(7 Marks)**

(c) Prove that $J_n(x) = \frac{1}{\pi}\int_0^\pi cos(n\theta - xsin\theta)d\theta$ where n is a positive integer. **(6 Marks)**

4. (a) If $v = (x^2 - 1)^n$, prove that $v_n = D^n v$ satisfies the Legendre's differential equation. Hence deduce the Rodrigue's formula for $P_n(x)$. **(7 Marks)**

(b) Prove that
i) $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$
ii) $xP_n'(x) - P_{n-1}'(x) = nP_n(x)$ **(7 Marks)**

(c) Prove that $\int_{-1}^{+1} P_n^2(x)dx = \frac{2}{2n+1}$ **(6 Marks)**

Contd.... 2

Part C

5. (a) Fit a parabola $y = a + bx + cx^2$ by the method of least squares to the following data:

x:	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y:	1.1	1.3	1.6	2.0	2.7	3.4	4.1

(7 Marks)

(b) In a partially destroyed laboratory record of correlation data, the following results only are available:

Variance of x is 9. Regression equations are $4x-5y+33 = 0, 20x - 9y = 107$. Calculate

i) the mean values of x and y

ii) standard deviation of y, and

iii) the coefficient of correlation between x and y. **(7 Marks)**

(c) A box A contains 2 white and 4 black balls. Another box B contains 5 white and 7 black balls. A ball is transferred from box A to box B. Then a ball is drawn from box B. Find the probability that it is white. **(6 Marks)**

6. (a) The probability distribution of a random variable X is given by the following table:

x_i:	-2	-1	0	1	2	3
$P(x_i)$:	0.1	k	0.2	$2k$	0.3	k

i) Find the value of k and calculate the mean and variance.

ii) Find $P(X > -1)$ **(7 Marks)**

(b) Given that 2% of the fuses manufactured by a firm are defective, find, by using Poisson distribution, the probability that a box containing 200 fuses has

i) no defective fuses

ii) 3 or more defective fuses

iii) at least one defective fuse. **(7 Marks)**

(c) In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for

i) 10 minutes or more

ii) less than 10 minutes

iii) between 10 minutes and 12 minutes. **(6 Marks)**

Part D

7. (a) Explain the following terms:

i) Type I and Type II errors

ii) Null hypothesis

iii) Level of significance. **(7 Marks)**

(b) A die was thrown 1200 times and the number 6 was obtained 236 times. Can the die be considered fair at 0.01 level of significance? **(7 Marks)**

(c) A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure:5,2,8,-1,3,0,6,-2,1,5,0,4 (in appropriate units). Can it be concluded that, on the whole the stimulus will change the blood pressure? Use $t_{0.05}(11) = 2.201$. **(6 Marks)**

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8. (a) The joint distribution of two random variables X and Y is given by the following table:

$X \setminus Y$	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Determine the marginal distribution of X and Y. Also, verify that X and Y are independent. **(7 Marks)**

(b) A student's study habits are as follows: If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night, he is 60% sure not to study the next night. In the long run how often does he study? **(7 Marks)**

(c) Define i) Transient state

ii) Recurrent state

iii) Absorbing state **(6 Marks)**

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NEW SCHEME

MAT41

USN

VIVEKANANDA COLLEGE OF ENGG & TECHNOLOGY LIBRARY

Fourth Semester B.E Degree Examination, July/August 2005

Engineering Mathematics IV

Common to all branches

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer any FIVE full questions choosing at least one question from each part.*
2. Statistical tables are allowed.

PART - A

1. (a) If $f(z) = u + iv$ is analytic, Prove that

$$\left[\frac{\partial}{\partial x} | f(z) |\right]^2 + \left[\frac{\partial}{\partial y} | f(z) |\right]^2 = | f'(z) |^2$$ **(7 Marks)**

(b) Give $u - v = (x - y)(x^2 + 4xy + y^2)$, find the analytic function $f(z) = u + iv$ **(7 Marks)**

(c) Find the bilinear transformation that maps the points $0, -i, -1$ of $Z-$ plane onto the points $i, 1, 0$ of W-plane respectively. **(6 Marks)**

2. (a) If a complex function $f(z)$ is analytic on and within a simple closed curve C then prove that $\oint_c f(z)dz = 0$ **(7 Marks)**

(b) Expand $f(z) = \frac{z}{(z-1)(z-3)}$ in power series valid for the regions i) $1 <| z |< 3$ ii) $| Z - 1 |< 2$ **(7 Marks)**

(c) Evaluate $\int_c \frac{Ze^z}{Z^2 - 1} dz$, where $C :| z |= 2$ **(6 Marks)**

PART - B

(a) Show that

i) $2nJ_n(x) = x[J_{n+1}(x) - J_{n-1}(x)]$

ii) $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ **(4+3 Marks)**

(b) Obtain the Jacobi's series

$cos(xsin\Phi) = J_0 + 2[J_2cos2\Phi + J_4cos4\Phi + ...]$

$sin(xsin\Phi) = 2J_1sin\Phi + 2J_3sin3\Phi + ...$ **(7 Marks)**

(c) Prove that $J_n(x) = \frac{1}{\pi}\int_0^{\pi} cos(n\Phi - xsin\Phi)d\Phi$ where n is a positive integer. **(6 Marks)**

4. (a) Obtain the series solution of the Legendre's differential equation $(1 - x^2)y'' - 2xy' + n(n + 1)y = 0$ **(7 Marks)**

Contd.... 2

(b) Establish the Rodrigue's formula

$$P_n(x) = \frac{1}{2^n \underline{|n}} \frac{d^n}{dx^n} (x^2 - 1)^n$$

(7 Marks)

(c) Show that $P_{2n}(0) = (-1)^n \frac{1.3.5...(2n-1)}{2.4.6...2n}$ and $P_{2n+1}(0) = 0$ **(6 Marks)**

PART - C

5. (a) Fit a curve of the form $y = a_0 + a_1 x + a_2 x^2$ to the data

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

by the method of least squares.

(7 Marks)

(b) In a partially destroyed lab record of analysis of correlation data, the following results are available. Variance of x is 9. Regression equations are $8x - 10y + 66 = 0$ and $40x - 18y - 214 = 0$.

Find $\overline{x}$, $\overline{y}, \sigma_y$ and correlation coefficient.

(7 Marks)

(c) The probabilities that A, B, C hit a target are respectively $\frac{1}{6}$, $\frac{1}{4}$ and $\frac{1}{3}$. Each shoots once at the target. Find the probability that exactly one of them hits the target. If only one hits the targent, what is the probability that it was A.

(6 Marks)

6. (a) The probability that a man aged 60 will live upto 70 is 0.65. Out of 10 men, now at the age of 60, find the probability that

i) at least 7 will live upto 70

ii) exactly 9 will live upto 70

iii) at most 9 will live upto 70

(7 Marks)

(b) A random variable X has the density function

$$p(x) = \begin{cases} kx^2 & for \ -3 \le x \le 3 \\ 0 & elsewhere \end{cases}$$

find k. Also find $P[x \le 2]$ and $P[x > 1]$

(7 Marks)

(c) The life of an electric bulb is a normal variate with mean life of 2040 hours and standard deviation of 60 hours. Find the probability that a randomly selected bulb will burn for

i) more than 2150 hours

ii) less than 1950 hours

Given $P[0 \le Z \le 1.83] = 0.4664$; $P[0 \le Z \le 1.33] = 0.4082]$ **(6 Marks)**

PART - D

7. (a) Explain :

i) Null hypothesis

ii) Type I and Type II errors

iii) Significance level

(7 Marks)

(b) A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis, at 5% level of significance, that the coin is unbiased **(7 Marks)**

Contd... 3

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(c) A sample of 12 measurements of the diameter of metal ball gave the mean $7.38mm$ with standard deviation $1.24m.m$ Find i) 95% and ii) 99% confidence limits for actual diameter. Given $t_{0.05}(11) = 2.20$ and $t_{0.01}(11) = 3.11$ **(6 Marks)**

8. (a) The joint probability distribution of two random variables X and Y is given below :

X \ Y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

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Find i) Marginal distributions of X and Y
ii) Covariance of X and Y **(7 Marks)**

(b) Three boys A, B and C are throwing a ball to each other. A always throws the ball to Band B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find the probability that A has the ball for the fourth throw. **(7 Marks)**

(c) Define :

i) Stochastic matrix

ii) Periodic state

iii) Absorbing state of a Markov chain **(6 Marks)**

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06MAT41

Fourth Semester B.E. Degree Examination, June / July 08

Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note : ***Answer any FIVE full questions, choosing at least two from each part.***

PART - A

1 a. Solve $\frac{dy}{dx} = x^2y - 1$ with y(0) = 1 using Taylor's series method and find y(0.1). Consider up to fourth degree terms. **(06 Marks)**

b. Use Runge – Kutta 4th order method to solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0) =1 and find y for x = 0.2 and 0.4. Take h = 0.2. **(07 Marks)**

c. Given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$, $y(0.1) = 1.1169$, $y(0.2) = 1.2773$, $y(0.3) = 1.5049$, find $y(0.4)$ accurate up to 3 decimal places using Milne's predictor – corrector method. **(07 Marks)**

2 a. If f(z) = u + i υ is analytic, show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2 + \left[\frac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$. **(06 Marks)**

b. Show that $u = e^{2x}(x\cos 2y - y\sin 2y)$ is harmonic. Find the analytic function f(z) = u + i v. **(07 Marks)**

c. Find the bilinear transformation that maps $Z_1 = i$, $Z_2 = 1$, $Z_3 = -1$ onto the points $W_1 = 1$, $W_2 = 0$, $W_3 = \infty$ respectively. Also find the image of $|Z|<1$ in w–plane under this transformation. **(07 Marks)**

3 a. If f(z) = u + I v is an analytic function and f′(z) is continuous at each point with in and on a closed curve C, then show that $\int_C f(z)dz = 0$. **(06 Marks)**

b. Expand $f(z) = \frac{z+1}{(z+2)(z+3)}$ in a Laurent's series valid for

i) $|z| > 3$ ii) $2 < |z| < 3$. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is the circle $|z| = 3$. **(07 Marks)**

4 a. Obtain a series solution for the differential equation : $y'' - xy' + y = 0$ **(06 Marks)**

b. Prove the Rodrigue's formula $P_n(x) = \frac{1}{2^n \lfloor n} \frac{d^n}{dx^n}\left[(x^2-1)^n\right]$. **(07 Marks)**

c. Express $x^3 + 2x^2 - x = 3$ in terms of Legendre polynomials. **(07 Marks)**

PART - B

5 a. Fit a 2nd degree polynomial of the form $y = a + bx + cx^2$ for the data : **(06 Marks)**

x	0	1	2	3	4	5
y	1	3	7	13	21	31

b. Find the coefficient of correlation, line of regression of x on y and line of regression of y on x ; given **(07 Marks)**

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

c. Three machines a, b, c produce respectively 50%, 30% and 20% of the items in a factory. The percentage of defective outputs of these machines are 3, 4 and 5 respectively. An item selected at random is found to be defective. Find the probability that it is from machine A. **(07 Marks)**

6 a. The probability that an individual suffers a bad reaction from an injection is 0.001. Find the probability that out of 2000 individuals, more than 2 will get a bad reaction. (06 Marks)

b. The sale per day in a shop is exponentially distributed with average sale amounting to Rs. 100 and net profit is 8%. Find the probability that the net profit exceeds Rs. 30 on a day. (07 Marks)

c. The life of an electric bulb is normally distributed with average life of 2000 hours and standard deviation of 60 hours. Out of 2500 such bulbs, find the number of bulbs that are likely to last between 1900 and 2100 hours. Given $P[0 \le z \le 1.67] = 0.4525$. (07 Marks)

7 a. The weights of 1500 ball bearings are normally distributed with a mean of 635 gms and standard deviation of 1.36 gms. If 300 samples of size 36 each are drawn from this population, find the expected mean and standard deviation of the sampling distribution of means, if sampling is done

i) With replacement ii) What out replacement. When sampling is done with replacement, how many samples will have their mean, greater than 635.5 gms. Given $P[0 \le z \le 2.203] = 0.4861$. (06 Marks)

b. Eleven students were given a test in statistics. They were provided additional coaching and then a second test of equal difficulty was held at the end of coaching. Marks scored by them in the two tests are given below :

Test – 1	23	20	19	21	18	20	18	17	23	16	19
Test – 2	24	19	22	18	20	22	20	20	23	20	17

Do the marks give evidence that the students have benefited by extra coaching? Given $t_{0.05}^{(\gamma=10)}$ is 2.228. Test the hypothesis at 5% level of significance. (07 Marks)

c. Fit a Poisson distribution to the following data and test for its goodness of fit at 5% level of significance.

x	0	1	2	3	4
f	419	352	154	56	19

Given $\chi^2_{0.05}$ for $\gamma = 3$ is 7.82. (07 Marks)

8 a. The joint probability distribution for two random variables X and Y is as given below.

X \ Y	-2	-1	4	5
1	0.1	0.2	0	0.3
2	0.2	0.1	0.1	0

Find the marginal distributions of X, Y. Also find the covariance of X and Y. (06 Marks)

b. Find the fixed probability vector of the regular stochastic matrix

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}.$$ (07 Marks)

c. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C. But C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball, find the probabilities that

i) A has the ball ii) B has the ball iii) C has the ball, for the fourth throw. (07 Marks)

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06MAT41

Fourth Semester B.E. Degree Examination, Dec.08 / Jan.09
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note : *Answer FIVE full questions choosing atleast two from each part.*

Part A

1 a. Find by Taylor's series method the value of y at $x = 0.1$ and $x = 0.2$ to five places of decimals from $\frac{dy}{dx} = x^2y - 1$, $y(0) = 1$ consider upto 4^{th} degree terms. **(06 Marks)**

b. Apply Runge-Kutta method to find an approximate value of y for $x = 0.2$ in steps of 0.1 of $\frac{dy}{dx} = x + y^2$, given that y = 1, when x = 0. **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(1+y)$ and y(1)=1, y(1.1)=1.233, y(1.2)=1.548, y(1.3)=1.979, evaluate y(1.4) by Adam's-Bashforth method. **(07 Marks)**

2 a. Derive Cauchy – Riemann equations in polar-form. **(06 Marks)**

b. Determine the analytic function, $f(z) = u + iv$, if

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}.$$ **(07 Marks)**

c. Discuss the transformation $w = e^z$. **(07 Marks)**

3 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Find the Taylor's expansion of $f(z) = \frac{2z^3 + 1}{z^2 + z}$ about the point $z = i$. **(07 Marks)**

c. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where C is the circle $|z| = 3$. **(07 Marks)**

4 a. Solve in series the equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$. **(06 Marks)**

b. Reduce the differential equation $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + k^2xy = 0$ to Bessel's equation. **(07 Marks)**

c. Derive the Rodrigue's formula, $P_n(x) = \frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2 - 1)^n$. **(07 Marks)**

Part B

5 a. Fit a second degree polynomial to the following data: **(06 Marks)**

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

b. The two regression equations of the variables x and y are
$x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$
Find i) mean of x's ii) mean of y's and iii) the correlation coefficient of x and y. **(07 Marks)**

c. State and prove Baye's theorem. **(07 Marks)**

6 a. The probability density function of a variate x is

x	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

i) Find k.

ii) Find P(x<4), and $P(3<x\leq 6)$. (06 Marks)

b. Derive mean and variance for the Poisson distribution. (07 Marks)

c. In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. if 60 hours. Estimate the number of bulbs likely to burn for

i) More than 2150 hours

ii) Less than 1950 hours and

iii) More than 1920 hours, but less than 2160 hours. (07 Marks)

7 a. In a city A, 20% of a random sample of 900 school boys has a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant? (06 Marks)

b. A machinist is making engine parts with axle diameter of 0.7 inch. A random sample of 10 parts shows mean diameter 0.742 inch with a S.D. of 0.04 inch. On the basis of this sample, would you say that the axle is inferior? (07 Marks)

c. A set of five similar coins is tossed 320 times and the result is:

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution. (07 Marks)

8 a. The joint distribution of two random variables x and y is given by the following table:

x \ y	2	3	4
1	0.06	0.15	0.09
2	0.14	0.35	0.21

Determine the marginal distribution of x and y. Also verify that x and y are stochastically independent. (06 Marks)

b. Find the fixed probability vector of the regular stochastic matrix,

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}.$$

(07 Marks)

c. Explain i) Transient state ii) Recurrent state iii) absorbing state of Markov chain. (07 Marks)

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Fourth Semester B.E. Degree Examination, June-July 2009

Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Solve $\frac{dy}{dx} = 2y + 3e^{\lambda}$, $y(0) = 0$. Using Taylor's series method an find y(0.1), y(0.2). **(06 Marks)**

b. Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ at $x = 0.2$ with step length $h = 0.2$. **(07 Marks)**

c. Use Milne's predictor –corrector method to find y at x = 0.8, given $\frac{dy}{dx} = x - y^2$ with,

X	0	0.2	0.4	0.6
Y	0	0.02	0.0795	0.1762

Apply corrector once. **(07 Marks)**

2 a. Find the analytic function $f(2) = u + iv$ if $v = e^x (x \sin y + y \cos y)$. **(06 Marks)**

b. Find the image of lines parallel to x – axis and lines parallel to y – axis under the transformation $w = z^2$. Draw neat sketch. **(07 Marks)**

c. Find the bilinear transformation that maps the points z = -1, j, 1 on to the points w = 1, j, -1. **(07 Marks)**

3 a. If f(z) is analytic within and on a simple closed curve C and 'a' is a point within 'C' then prove that $f(a) = \frac{1}{2\pi j}\int_c \frac{f(z)}{z-a}dz$. **(06 Marks)**

b. State Cauchy's residue theorem. Hence or otherwise evaluate –

$\int_c \frac{e^{2z}}{(z+2)(z+4)(z+7)}dz$ for 'C' as $|Z| = 3$. **(07 Marks)**

c. Find the Taylor's series expansion of $f(z) = \frac{1}{(z+1)^2}$ about the point z = -i. **(07 Marks)**

4 a. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\sin x$. **(06 Marks)**

b. Express polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. **(07 Marks)**

c. Compute P_0, P_1, P_2, P_3, P_4 using Rodrigue's formula. **(07 Marks)**

PART – B

5 a. Fit a parabola $y = a + bx + cx^2$, given the data : **(06 Marks)**

x	−3	−2	−1	0	1	2	3
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58

b. Obtain the coefficient of correlation and the liens of regression if : **(07 Marks)**

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

c. A tea set has four sets of cups and saucers. Two of these sets are of one colour and the other two sets are of different colours. (totally three colours). If the cups are placed randomly on saucers, what is the probability that no cup is on a saucer of same colour. **(07 Marks)**

6 a. Define i) Random variable ii) Discrete probability distribution with an example. (06 Marks)
b. The probability that a man aged 60 will live up to 70 is 0.65. What is the probability that out of 10 men, now aged 60, i) exactly 9, ii) at the most 9 iii) at least 7, will live up to the age of 70 years. (07 Marks)
c. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation, given that A(0.5) = 0.19 and A(1.4) = 0.42. (07 Marks)

7 a. Find the probability that in 100 tosses of a fair coin between 45% and 55% of the outcomes are heads. (06 Marks)
b. A mechanist is making engine parts with axle diameter of 0.7 inches. A random sample of 10 parts showed a mean of 0.472 inches with a standard deviation of 0.04 inches. On the basis of this sample, can it be concluded that the work is inferior at 5% level of significance. (07 Marks)
c. For the following data test the hypothesis that the accidents are uniformly distributed over all the days of the week for 99% confidence.

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accidents	14	16	8	12	11	9	14	84

(07 Marks)

8 a. Find the –
Marginal distribution of x
Marginal distribution of y
Cov (x, y) if the joint pdf of x and y is

x \ y	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

(06 Marks)

b. Find the fixed probability vector of regular stochastic matrix

$$A = \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}.$$ (07 Marks)

c. A company executive changes his car every year. If he has a car of make A, he changes over to make B. from make B he changes over to make C. if he has car 'C' then he gives equal preference to change over to make A or make B car. If he had a car of make C in year 2008 find the probability that he will have a car of i) make A in 2010, ii) make 'C' in 2010. (07 Marks)

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06MAT41

Fourth Semester B.E. Degree Examination, Dec.09/Jan.10
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Employ Taylor's series method to find an approximate solution correct to fourth decimal places for the following initial value problem at $x = 0.1$, $dy/dx = x - y^2$, $y(0)=1$. **(06 Marks)**

b. Using modified Euler's method to find $y(0.1)$ given $dy/dx = x^2 + y$, $y(0) = 1$ by taking $h=0.05$. Perform two iterations in each step. **(07 Marks)**

c. If $dy/dx = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.04$ and $y(0.3)=2.09$ find $y(0.4)$ correct to four decimal places. By using Milne's predictor-corrector method (Use corrector formula twice). **(07 Marks)**

2 a. Derive Cauchy-Riemann equations in Cartesian form. **(06 Marks)**

b. Find the analytic function $f(z) = u+iv$ whose real part is $e^{-x}(x\cos y + y\sin y)$. **(07 Marks)**

c. Find the bilinear transformation which maps the points $Z=0, i, \infty$ onto the points $w = 1, -i, -1$ respectively. Find the invariant points. **(07 Marks)**

3 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Expand $f(z) = \dfrac{1}{(z-1)(z-2)}$ in terms of Laurent's series valid in the regions i) $|z-1| < 1$ ii) $|z-1| > 1$. **(07 Marks)**

c. Evaluate $\int_c \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}$ using Cauchy's Residues theorem where c is the circle $|z| = 3$. **(07 Marks)**

4 a. Solve in series the equation $x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + xy = 0$ **(06 Marks)**

b. Solve Bessel's differential equation leading to $J_n(x)$. **(07 Marks)**

c. Express $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials. **(07 Marks)**

PART – B

5 a. The pressure and volume of a gas are related by the equation $PV^v = K$, where v and K being constants. Fit this equation to the following set of observations. **(06 Marks)**

P (kg/cm^2)	0.5	1.0	1.5	2.0	2.5	3.0
V (litre)	1.62	1.00	0.75	0.62	0.52	0.46

b. Find the correlation coefficient and the regression lines of y on x and x on y for the following data: **(07 Marks)**

x	1	2	3	4	5
y	2	5	3	8	7

c. State and prove Baye's theorem. **(07 Marks)**

6 a. The probability density function of a variate X is

X:	0	1	2	3	4	5	6
P(X):	k	3k	5k	7k	9k	11k	13k

Find i) k ii) $P(X \geq 5)$ iii) $P(3 < X \leq 6)$ (06 Mark

b. The number of telephone lines busy at an instant of time is a binomial variate wi probability 0.1 that a line is busy. If 10 lines are choosen at random, what is the probabili that i) no line is busy ii) at least 5 lines are busy iii) at most 3 lines are busy. (07 Mark

c. Obtain the mean and standard deviation of the normal distribution. (07 Mark

7 a. Explain the following terms:
   i) Null hypothesis
   ii) Confidence limits
   iii) Type I & Type II errors. (06 Mark

b. A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On tl assumption of random throwing, do the data indicate that the die is biased? (07 Mark

c. The nine items of a sample have the following values: 45, 47, 50, 52, 48, 47, 49, 53, 51. Does the mean of these differ significantly from the assumed mean of 47.5? (Given ... 8 df = 2.31). (07 Mark

8 a. The joint probability distribution of two random variables X and Y are given below.

X \ Y	-3	2	4
1	0.1	0.2	0.2
2	0.3	0.1	0.1

Determine i) E(X) and E(Y) ii) E(XY) iii) COU (X,Y) (06 Mark

b. Every year, a man trades his car for a new car. If he has a Maruti, he trades it for a Ambassador. If he has an Ambassador, he trades it for a Santro. However, if he has a Santro he is just as likely to trade if for a new Santro as to trade if for Maruti or an Ambassador. In 2000, he bought his first car, which was Santro. Find the probability that he has i) 2002 Santro ii) 2002 Maruti. (07 Mark

c. Define stochastic matrix. Find the unique fixed probability vector for the regular st... matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$ (07 Mark

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USN

Fourth Semester B.E. Degree Examination, May/June 2010

Engineering Mathematics - IV

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting at least TWO questions from each part.***
2. Use of statistical tables is permitted.

PART – A

1 a. Find the y(0.1) correct to 6 decimal places by Taylor series method when $dy/dx = xy + 1$, $y(0) = 1.0$. (Consider upto 4^{th} degree term). **(06 Marks)**

b. Using Runge-Kutta method of order 4, compute y(0.2) for the equation, $y' = y - \frac{2x}{y}$, $y(0) = 1.0$ (Take $h = 0.2$). **(07 Marks)**

c. Given that $y' = x^2(1+y)$ and $y(1) = 1.0$, $y(1.1) = 1.233$, $y(1.2) = 1.548$ and $y(1.3) = 1.979$, compute y(1.4) by Adams-Bashforth method. Apply correct formula twice. **(07 Marks)**

2 a. Show that Z^n is analytic. Hence find its derivative. **(06 Marks)**

b. Find a bilinear transformation which maps the points 0, 1, i in the Z-plane onto $1 + i$, $-i$, $2 - i$ in the W plane. **(07 Marks)**

c. Find the analytic function $u + iv$, where u is given to be $u = e^x [(x^2 - y^2)\cos y - 2xy \sin y]$. **(07 Marks)**

3 a. Derive Couchy's integral formula in the form

$$f(a) = \frac{1}{2\pi i}\int_c \frac{f(z)dz}{z-a}$$ **(06 Marks)**

b. Expand $f(z) = \frac{7z^2 + 9z - 18}{z^3 - 9z}$ in the Laurent series that is valid for
i) $|z| > 3$ ii) $0 < |z - 3| < 3$. **(07 Marks)**

c. Evaluate $\int_c \tan z\, dz$, where c is $|z| = 2.5$ **(07 Marks)**

4 a. Find the series solution of $\frac{d^2y}{dx^2} + xy = 0$. **(06 Marks)**

b. Express $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials. **(07 Marks)**

c. Reduce the differential equation $x\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + k^2xy = 0$ to Bessel's equation. Obtain the solution. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – B

5 a. Fit a curve of the form $y = ab^x$ for the data given below: (06 Marks)

x :	2	4	6	8	10	12
y :	1.8	1.5	1.4	1.1	1.1	0.9

b. Find the coefficient of correlation for the following data: (07 Marks)

x :	55	56	58	59	60	60	62
y :	35	38	39	38	44	43	45

c. In a certain college 25% of boys and 10% of girls are studying mathematics. The girls constitute 60% of the student body.
   i) What is the probability that mathematics is being studied?
   ii) If a student is selected a random and is found to be studying mathematics, find the probability that the student is a girl. (07 Marks)

6 a. Suppose a random variable X takes the values –3, –1, 2 and 5 with respective probabilities $\frac{2k-3}{10}, \frac{k-2}{10}, \frac{k-1}{10}, \frac{k+1}{10}$. Find the value of k and i) find $P[-3 < X < 4]$ and ii) $P[X \le 2]$. (06 Marks)

b. Suppose that the student IQ scores form a normal distribution with mean 100 and standard deviation 20. Find the percentage of students whose i) score is less than 80 ii) score falls between 90 and 140, iii) Score more than 120. (07 Marks)

c. Obtain mean and variance of binomial distribution function. (07 Marks)

7 a. A sample of 1000 days is taken from meteorological records of a certain district and 120 of them are found to be foggy. What are the probable 99% confidence limits to the proportion of foggy days in the district? (06 Marks)

b. The following table gives the number of bus accidents that occurred during the various days of the week. Find whether the accidents are uniformly distributed over the week, using χ^2 test. (07 Marks)

Days	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accidents	14	16	8	12	11	9	14	84

c. The life X of certain computer is approximately normally distributed with mean 800 hours and standard deviation 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the hypothesis that $\mu = 800$ hours against the alternate hypothesis $\mu \neq 800$ hours at i) 0.5% and 1% level of significance. (07 Marks)

8 a. A fair coin is tossed 4 times. Let X denote the number of heads occurring and let Y denote the longest string of heads occurring. Find the joint distribution function of X and Y. (06 Marks)

b. A man's gambling luck follows a pattern. If he wins a game the probability of winning the next game is 0.6. However, if he loses a game, the probability of losing the next game is 0.7. There is an even chance that he wins the first game.
i) Find the transition matrix of the Markov process. ii) Find the probability that he wins the third game. iii) Find out how often, in the long run, he wins. (07 Marks)

c. Explain: i) Transient state ii) Absorbing state and iii) Recurrent state by means of an example each. (07 Marks)

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MAT41

NEW SCHEME

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Fourth Semester B.E Degree Examination, July/August 2004

Engineering Mathematics IV

Common to all branches

(New Scheme)

Time: 3 hrs.] [Max.Marks : 100

Note: *1. Answer FIVE full questions choosing at least one from each part.*
2. All questions carry equal marks.
3. Statistical tables are allowed.

Part A

1. (a) Derive Cauchy-Reimann equations in Cartesian form.

(b) If $f(z)$ is an analytic function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) | f(z) |^2 = 4 | f'(z) |^2$$

Find the images of

i) $x - y = 1$ and

ii) $x^2 - y^2 = 1$

under the transformation $w = z^2$. (7+7+6=20 Marks)

2. (a) State and prove Cauchy's integral formula.

(b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in terms of Laurent's series valid in the regions

i) $| z - 1 | < 1$;

ii) $| z - 1 | > 1$

(c) Evaluate

$$\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$

where c is $| z | = 3$ using Cauchy's residue theorem. (7+7+6=20 Marks)

Part B

3. (a) Solve Bessel's differential equation leading to $J_n(x)$.

(b) Prove that $\int_0^1 x J_n^2(\alpha x) dx = \frac{1}{2}[J_n^1(\alpha)]^2$, where α is the root of the equation $J_n(x) = 0$

(c) Show that

i) $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \sin x$ ii) $J_{\frac{-1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \cos x$ (7+7+6=20 Marks)

Contd.... 2

4. (a) Prove that $(1-2xt+t^2)^{\frac{-1}{2}} = \sum_{n=-\infty}^{\infty} t^n P_n(x)$

(b) Show that

i) $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$

ii) $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$

(c) Express $x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

(7+7+6=20 Marks)

PART C

5. (a) Fit a parabola to the data:

x:	1.0	1.5	2.0	2.5	3.0	3.5	4.0
y:	1.1.	1.3	1.6	2.0	2.7	3.4	4.1

(b) Define coefficient of correlation. If θ is the angle between two regression lines, show that

$$tan\theta = \frac{1-r^2}{r} \frac{\sigma x\ \sigma y}{\sigma x^2+\sigma y^2},$$

and explain the significance when $r = 0$.

(c) If A and B are any two arbitrary events, prove that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(7+7+6=20 Marks)

6. (a) Define random variable. A random variable $X(= x)$ has the following probability distribution:

x:	0	1	2	3	4	5	6	7
p(x):	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find i) k ii) $p(X < 6)$ iii) $p(X \geq 6)$

(b) If 10 percent of the rivets produced by a machine are defective, find the probability that out of 12 rivets chosen at random,

i) exactly 2 will be defective

ii) at least 2 will be defective

iii) none will be defective

(c) Show that mean and standard deviation of exponential distribution are equal.

(7+7+6=20 Marks)

PART D

7. (a) Explain the following terms:

i) Null hypothesis

ii) Confidence limits

iii) Type I and Type II errors.

(b) What do you mean by level of significance? A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate that the die is unbiased?

Contd..

(c) Eleven school boys were given a test in drawing. They were given a month's further tution and a second test of equal difficulty was held at the end of it. Do the marks give evidence that the students have benefitted by extra coaching? (Given $t_{0.05}$ (for $\gamma = 10$) = 2.228.

Boys :	1	2	3	4	5	6	7	8	9	10	11
Marks (I Test):	23	20	19	21	18	20	18	17	23	16	19
Marks (II Test):	24	19	22	18	20	22	20	20	23	20	17

(7+7+6=20 Marks)

8. (a) The joint distribution of two random variables X and Y is given by the following table:

X \ Y	−4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Determine

i) the marginal distributions of X and Y

ii) $E(X)$ and $E(Y)$

iii) Are X and Y independent random variables?

(b) Every year, a man trades his car for a new car. If he has a Maruti, he trades it for an Ambassador. If he has an Ambassador, he trades it for a Santro. However, if he has a Santro, he is just as likely to trade it for a new Santro as to trade it for a Maruti or an Ambassador. In 2000, he bought his first car, which was a Santro. Find the probability that he has

i) 2002 Santro ii) 2002 Maruti.

(c) Define stochastic matrix. Find the unique fixed probability vector for the regular stochastic matrix

$$\begin{pmatrix} 0 & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

(7+7+6=20 Marks)

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